Mathematics

(041)

Space for writing Question Number

SECTION-A

Q.1: (A) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

Q.2: (D) $[2,\infty)$

Q.3: (D) $2\int_0^a f(x)\,dx$.

Q.4: (D) 8.

Q.5: (B) $\left[\frac{-\pi}{2},0\right]$

Q.6: (B) $\frac{\pi}{3}$ only.

Q.7: (D) $\frac{1-P(E\cup F)}{P(\bar{F})}$

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Q.8 : (C) Null matrix

Q.9 : (C) $x = 3t+4, \quad y = t-3, \quad z = 2t+7$

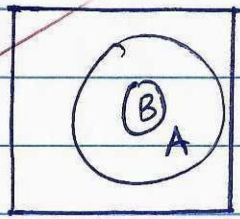
Q.10 : (B) Bina

Q.11 : (C) 1 cm/s.

Q.12 : (D) $\frac{2\pi}{3}$

Q.13 : (C) $(1, -5, -6)$.

Q.14 : (B)



Q.15 : (D) $\int_0^4 \sqrt{y}\, dy$

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Q.16 : (C) The objective function maximizes the combined profit earned from selling x & y.

Q.17 (B) AB = BA.

Q.18 : (B) 1.

Q.19 : (D) Assertion (A) is false but Reason (R) is true.

Q.20 : (D) Assertion (A) is false but Reason (R) is true.

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SECTION - D

Q-32: Required shaded area = Area of region $\overset{E}{\bullet}AB$ + Area of region $\overset{E}{\bullet}CD$

$$= \left| \int_{-2}^{-2/5} (5x+2)\,dx \right| + \int_{-2/5}^{2} (5x+2)\,dx$$

$$= \left| \left[\frac{5}{2}x^2 + 2x \right]_{-2}^{-2/5} \right| + \left[\frac{5}{2}x^2 + 2x \right]_{-2/5}^{2}$$

$$= \left| \frac{2}{5} + 2\left(\frac{-2}{5}\right) - 10 + 4 \right| + \left[10 + 4 - \frac{2}{5} + \frac{4}{5} \right]$$

$$= \frac{32}{5} + \frac{72}{5}$$

$$= \frac{104}{5} \text{ sq. units}$$

(Graph on page - 19).

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Q.33 :

let $I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} \, dx$

~~let~~ $\cancel{\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{(x^2+1)}{(x+2)(x^2+1)} + \frac{x}{x}}$.

let $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+c}{x^2+1}$

$$x^2+x+1 = A(x^2+1) + (Bx+c)(x+2)$$

$$x^2+x+1 = Ax^2 + A + Bx^2 + 2Bx + cx + 2c$$

$$x^2+x+1 = (A+B)x^2 + (2B+c)x + (2c+A).$$

On comparing LHS and RHS :-

$A+B = 1$ — (1) $\quad 2B+c = 1$ — (2) $\quad 2c+A = 1.$ — (3)

$B = 1-A$

Putting $B = 1-A$ in (2) : $2(1-A)+c = 1$

$$2-2A+c = 1$$

$$-2A+c = -1. \quad \text{— (4)}$$

Multiplying (4) with 2 and equating with (3) :

$$-4A + 2C = -2$$
$$\cancel{2C}\ A + 2C = 1$$
$$- \quad - \quad -$$
$$-5A = -3$$
$$A = \frac{3}{5}$$

$$B = 1 - A = 1 - \frac{3}{5} = \frac{2}{5}.$$

$$C = 1 - 2B = 1 - \frac{4}{5} = \frac{1}{5}.$$

Putting A, B & c in I :

$$I = \frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{2x+1}{x^2+1}\,dx$$

$$= \frac{3}{5}\log|x+2| + \frac{1}{5}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{dx}{x^2+1}$$

$$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + c$$

$\therefore$ $$\int \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + c$$

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Q. 34: (a) let $l_1 : \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$

$l_2 : \frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$

Expressing l_1 & l_2 in vector form of equations:-

$\vec{r_1} \Rightarrow (-\hat{i} + \hat{j} + 9\hat{k}) + \lambda(2\hat{i} + \hat{j} - 3\hat{k})$

$\vec{r_2} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \mu(2\hat{i} - 7\hat{j} + 5\hat{k})$

Comparing $\vec{r_1}$ & $\vec{r_2}$ with $\vec{r} = \vec{a} + k\vec{b}$'s

$\vec{a_1} = -\hat{i} + \hat{j} + 9\hat{k}$ $\quad$ $\vec{b_1} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\vec{a_2} = 3\hat{i} - 15\hat{j} + 9\hat{k}$. $\quad$ $\vec{b_2} = 2\hat{i} - 7\hat{j} + 5\hat{k}$.

$\vec{a_2} - \vec{a_1} = 4\hat{i} - 16\hat{j} + 0\hat{k}$.

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} = \hat{i}(5-21) - \hat{j}(10+6) + \hat{k}(-14-2)$$

$= -16\hat{i} - 16\hat{j} - 16\hat{k}$.

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$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-16)^2 + (-16)^2 + (-16)^2}$$
$$= \sqrt{3(16)^2}$$
$$= 16\sqrt{3}.$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (4\hat{i} - 16\hat{j} + 0\hat{k}) \cdot (-16\hat{i} - 16\hat{j} - 16\hat{k})$$
$$= -64 + 256 + 0$$
$$= 192.$$

$\therefore$ shortest distance between the given lines $= \left| \dfrac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$$= \frac{\cancel{192}^{\cancel{24}\ 12}}{\cancel{16}\sqrt{3}_{2}}$$

$$= \frac{12\sqrt{3}}{3}$$

$$= 4\sqrt{3} \text{ units.}$$

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Q.35: (a)

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$= 8I.$$

$\therefore AB = 8I \Rightarrow B^{-1} = \frac{1}{8}A.$ — (1)

The given system of linear equations can be represented in the following manner:-

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$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$BX = C$$

$$X = B^{-1}C$$

$$= \frac{1}{8} AC \qquad \text{(From (1)).}$$

$$= \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -16 + 36 + 4 \\ -28 + 9 + 3 \\ 20 - 27 - 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

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$$X = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \Rightarrow x = 3,\ y = -2,\ z = -1.$$

Ans : $x = 3,\ y = -2,\ z = -1.$

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SECTION-E

Q.36: $S = \{S_1, S_2, S_3, S_4\}$, $J = \{J_1, J_2, J_3\}$

No. of elements in $S = n(S) = 4$

No. of elements in $J = n(J) = 3$.

(i) No. of relations from S to J $= 2^{n(S) \times n(J)} = 2^{4\times 3} = 2^{12}$.

Ans: There can be 2^{12} relations from S to J.

(ii) $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$.

The given function f is not one-one / injective as the images of two elements under f i.e. S_2 and S_3 is the same i.e. J_2, which implies that every element of domain(S) does not have a distinct image in codomain (J).

∴ Since f is not injective, it cannot be bijective, even though it is surjective as every element of codomain has a preimage in Domain.

Ans: No, f is not bijective as it is not injective.

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ANSWER

(iii)

~~$R_1 = \{(S_1, S_2), (S_2, S_4)\}$~~

~~Ordered pairs to be added for the function to be reflexive but not symmetric are :~~

~~$(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$.~~

~~R_1 would hence become :~~

~~$R_1 = \{(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)\}$~~

(a) No. of one-one functions from S to T $= {}^{n(T)}P_{n(S)}$

Since $n(T) < n(S)$ $[3 < 4]$, no one-one functions are possible from S to T.

<u>Ans</u> : 0 one-one functions are possible from S to T.

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Q. 37

let E_1, E_2 and E_3 denote the events of Amber, Bonzi and Comet manufacturing the cars. let A denote the event of selecting an electric car manufactured.

$P(E_1) = 60\% = \frac{6}{10}$ $\quad$ $P(A|E_1) = 20\% = \frac{20}{100}$

$P(E_2) = 30\% = \frac{3}{10}$ $\quad$ $P(A|E_2) = 10\% = \frac{10}{100}$

$P(E_3) = 10\% = \frac{1}{10}$ $\quad$ $P(A|E_3) = 5\% = \frac{5}{100}$.

(i) (a) Probability a randomly selected car is an electric car

$= P(A)$

$= P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)$

$= \frac{6}{10} \cdot \frac{20}{100} + \frac{3}{10} \cdot \frac{10}{100} + \frac{1}{10} \cdot \frac{5}{100}$

$= \frac{120 + 30 + 5}{1000}$

$= \frac{155}{1000} = 0.155$

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(ii) Probability a randomly selected car which is electric ~~and~~ is manufactured by comet $= P(E_3 | A)$

$$= \frac{P(E_3) \cdot P(A|E_3)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)}$$

$$= \frac{\frac{1}{10} \cdot \frac{5}{100}}{\frac{155}{1000}}$$

$$= \frac{5}{155}$$

$$= \frac{1}{31} \quad \text{(Ans)}$$

(iii) Probability a randomly selected car which is electric was manufactured by Amber or Bonzei $= 1 - P(E_3 | A)$

$$= 1 - \frac{1}{31} = \frac{30}{31} \quad \text{(Ans)}$$

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Q: 38 :

$f(x) = e^x \sin x$

(i)

$f'(x) = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$.

Put $f'(x) = 0$ for critical points :

$e^x(\sin x + \cos x) = 0$.

Since in $[0, \pi]$, $e^x \neq 0$ for any $x \in [0, \pi]$,

$\Rightarrow \sin x + \cos x = 0$

$\Rightarrow \sin x = -\cos x$

$\Rightarrow -\tan x = 1$

$\Rightarrow \tan x = -1$

$\Rightarrow x = \frac{3\pi}{4}$.

$f''(x)$

$= e^x(\sin x + \cos x)$

$+ e^x(\cos x - \sin x)$

Interval	Sign of $f'(x)$	Increasing / Decreasing
$[0, \frac{3\pi}{4}]$	+	Increasing
$[\frac{3\pi}{4}, \pi]$	−	Decreasing.

Answer : $f(x)$ is increasing in $[0, \frac{3\pi}{4}]$ as $f'(x) > 0$ in this interval and $f(x)$ is decreasing in $[\frac{3\pi}{4}, \pi]$ as $f'(x) < 0$ in this interval.

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(ii) let us evaluate $f''(x)$ to check for local maxima / minima / inflection point

$f'(x) = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$.

$f''(x) = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$.

$f''\left(\frac{3\pi}{4}\right) = e^{3\pi/4}\left(\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4}\right) + e^{3\pi/4}\left(\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4}\right)$

$= e^{3\pi/4}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) + e^{3\pi/4}\left(\frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$

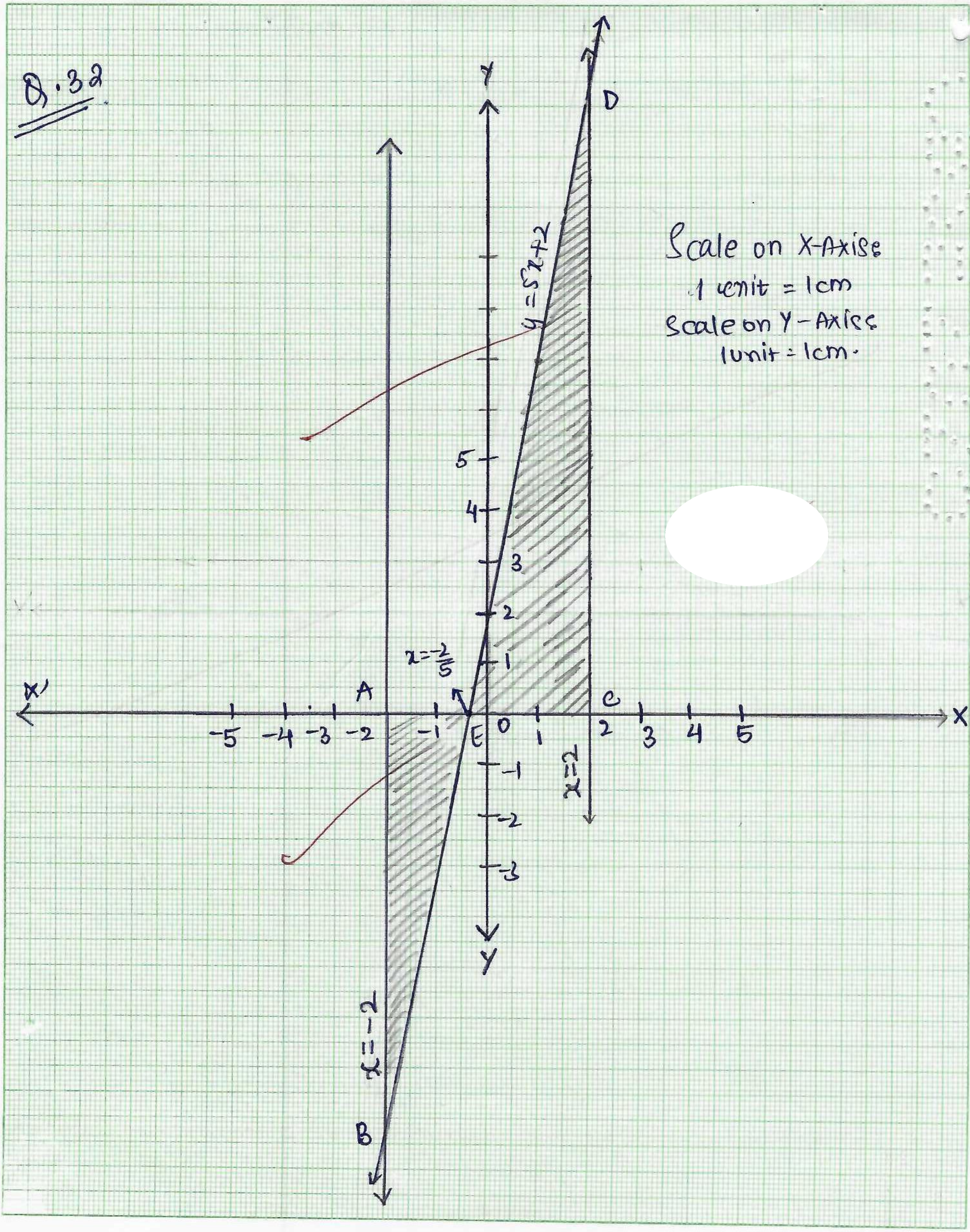
$= e^{3\pi/4}(-\sqrt{2})$.

$= -\sqrt{2}e^{3\pi/4} < 0$

$\Rightarrow x = \frac{3\pi}{4}$ is a point of local maximum.

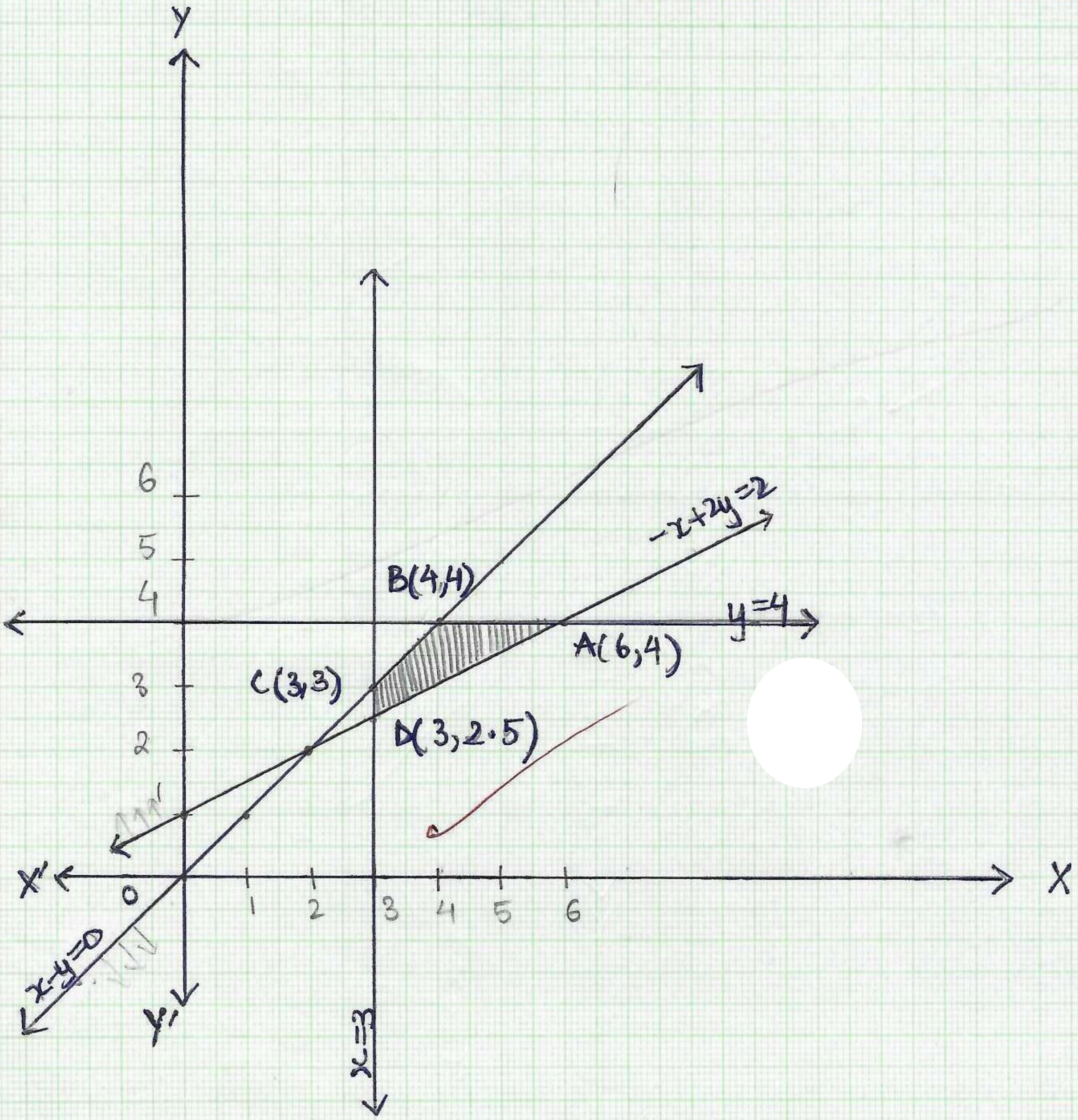
Answer: As $f''(x) < 0$ at $x = \frac{3\pi}{4}$, it is a point of local maximum.

Q.32



Scale on X-Axise
1 unit = 1cm
Scale on Y-Axise
1unit = 1cm.

Q.28.



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SECTION E

Q. 26 : (b) $(1+x^2)\dfrac{dy}{dx} + 2xy = 4x^2$

$$\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}.$$

This is a linear differential eq^n of the form $\dfrac{dy}{dx} + Py = Q$, where $P = \dfrac{2x}{1+x^2}$ and $Q = \dfrac{4x^2}{1+x^2}$.

Integrating factor (IF) $= e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log|1+x^2|}$
$= (1+x^2)$.

Solution of the differential equation is given by:

$$y(IF) = \int (IF)\,Q\,dx + c$$

$$y(1+x^2) = \int (1+x^2)\left(\frac{4x^2}{1+x^2}\right)dx + c$$

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$y(1+x^2) = \int 4x^2\,dx + c$

$\therefore y(1+x^2) = \frac{4}{3}x^3 + c$

Ans: The required general solution of the differential equation is $\therefore$ $y(1+x^2) = \frac{4}{3}x^3 + c$.

Q.27: $m R n$ if and only if m is a multiple of n, $m, n \in N$.

$\Rightarrow R = \{(m,n) : m = kn,\ m, n, k \in N\}$.

Reflexive: Since every natural number is a multiple of itself, $(m,m) \in R$ is true $\forall\ m \in N$.

This holds true as $m = km$, where $k = 1$.

$\therefore$ R is reflexive.

Symmetric: let $(m,n) \in R$, $m, n \in N$.

let $m = 2$ and $n = 1$.

As $m = 2 \times n$, $(2,1) \in R$.

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But $(1,2) \notin R$ as 1 is not a multiple of 2 and cannot be expressed in the form of $1 = 2k$, where k is a natural number. $\Rightarrow (n,m) \notin R$ for $n=1$ and $m=2$.

$\therefore$ R is not symmetric

(As for all $(m,n) \in R$, $(n,m) \in R$ is not true).

Transitive: let (m,n) and $(n,p) \in R$, $m,n,p = N$.

$\Rightarrow$ m is a multiple of n and

n is a multiple of p

$\Rightarrow m = k_1 n$ and $n = k_2 p$, $k_1, k_2 \in N$.

$\Rightarrow n = \frac{m}{k_1}$ and $n = k_2 p$

$\Rightarrow \frac{m}{k_1} = k_2 p$

$\Rightarrow m = (k_1 k_2)p$, $(k_1 k_2) \in N$.

$\Rightarrow$ m is a multiple of p.

$\Rightarrow (m,p) \in R$.

$\therefore$ R is transitive.

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Q. 288 $Z = x - 5y$ (to be minimized), $x, y \geq 0$.

Constraints:

(i) $x - y \geq 0$ (ii) $-x + 2y \geq 2$ (iii) $x \geq 3$ (iv) $y \leq 4$.

$\Rightarrow x - y = 0$

x	0	1
y	0	1

$\Rightarrow -x + 2y = 2$

x	0	2
y	1	2

The graph of the constraints has been plotted on page page 21.
The feasible region is bounded, hence minimum of z exists and is defined.

Corner Point	$Z = x - 5y$
A (6, 4)	−14
B (4, 4)	−16 → min.
C (3, 3)	−12
D (3, 2.5)	1

6 − 20

4 − 20

$\left(\frac{2}{2}\right) = 1$

Answer: Minimum value of z is −16 at corner point B(4, 4).

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Q.29: (b)

Given: $x\sqrt{1+y} + y\sqrt{1+x} = 0$.

To Prove: $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

Proof: $x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring both sides:

$$x^2(1+y) = y^2(1+x)$$

$$x^2 + x^2y = y^2 + xy^2$$

$$x^2 - y^2 = xy^2 - x^2y$$

$$x^2 - y^2 = xy(y-x)$$

$$(x-y)(x+y) = xy(y-x)$$

$$x+y = -xy$$

$$y + xy = -x$$

$$y(1+x) = -x$$

$$y = \frac{-x}{1+x}$$

PTO →.

$$y = \frac{-x}{1+x}$$

Differentiating both sides w.r.t x:

06

$$\frac{dy}{dx} = \frac{-(1+x) - (-x)}{(1+x)^2}$$

$$= \frac{-1-x+x}{(1+x)^2}$$

$$= \frac{-1}{(1+x)^2}$$

Hence, proved.

Q.30: (a)

$$P(2) = \frac{3}{10}.$$

$$P(\{1,3,4,5,6\}) = 1 - \frac{3}{10} = \frac{7}{10}$$

$$P(1) = P(3) = P(4) = P(5) = P(6) = \frac{1}{5} \times \frac{7}{10} = \frac{7}{50}.$$

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$\frac{9}{100}$ $\frac{21 \times 3}{50 \times 5}$ $\frac{42}{100}$ 25 × 49 = 225 + 1000 = 1225 $\frac{49}{100}$ 77 × 11 = 77 + 770 = 847

let X denote the number of times number 2 appear on the throw of a dice twice. X is a random variable and can take the values 0, 1 or 2.

favourable outcomes when 2 appear once or twice :-

$\{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (2,1), (2,3), (2,4), (2,5), (2,6)\}$.

n = no. of outcomes when 2 appear atleast once = 11.

Total outcomes on the throw of die twice = 36.

$P(X=0) = P(\text{not getting a two even once}) = \cancel{\frac{36-11}{36}} \cdot \cancel{\frac{25}{36}}$ $\cancel{\frac{7}{10} \times \frac{7}{10}} =$

$25 \times \frac{7}{50} \times \frac{7}{50}$

$P(X=1) = P(\{(1,2), (3,2), (4,2), (5,2), (6,2), (2,1), (2,3), (2,4), (2,5), (2,6)\})$

$= \cancel{\frac{10}{36}}$. $10 \times \frac{7}{50} \times \frac{3}{10}$

$P(X=2) = P(\{(2,2)\}) = \frac{3}{10} \times \frac{3}{10}$

Probability distribution of X :

X	0	1	2
P(X)	$\frac{49}{100}$	$\frac{42}{100}$	$\frac{9}{100}$

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$\therefore$ Mean no. of times 2 appear on the dice if thrown twice

$= E(X) = \sum P(x).x$

$= 0.\frac{49}{100} + 1.\frac{42}{100} + 2.\frac{9}{100}$

$= \frac{0+42+18}{100}$

$= \frac{60}{100}$

$= 0.6.$

Ans : $E(X) = 0.6$ or $\frac{3}{5}$.

Q.31: let $I = \int \frac{1}{x}\sqrt{\frac{x+a}{x-a}}\, dx$

$= \int \frac{1}{x}\sqrt{\frac{x+a}{x-a} \times \frac{x+a}{x+a}}\, dx$

$= \int \frac{1}{x}\sqrt{\frac{(x+a)^2}{x^2-a^2}}\, dx$

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$$= \int \frac{1}{x} \frac{(x+a)}{\sqrt{x^2-a^2}}\, dx$$

$$= \int \frac{x}{x\sqrt{x^2-a^2}}\, dx + \int \frac{a\, dx}{x\sqrt{x^2-a^2}}$$

$$= I_1 + I_2 .$$

$$I_1 = \int \frac{x}{x\sqrt{x^2-a^2}}\, dx = \int \frac{dx}{\sqrt{x^2-a^2}} = \log \left| x + \sqrt{x^2-a^2} \right| + c_1 .$$

$$I_2 = \int \frac{a\,dx}{x\sqrt{x^2-a^2}}$$

let $x = \frac{a}{t} \Rightarrow dx = \frac{-a}{t^2}\, dt .$

$$I_2 = \int \frac{a\left(\frac{-a}{t^2}\right) dt}{\frac{a}{t}\sqrt{\frac{a^2}{t^2} - a^2}} = \int \frac{\left(-\frac{a^2}{t^2}\right) dt}{\frac{a}{t}\sqrt{\frac{a^2-a^2t^2}{t^2}}}$$

PTO →.

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$$= \int \frac{\left(-\frac{a^2}{t^2}\right) dt}{\frac{a^2}{t^2}\sqrt{1-t^2}} = -\int \frac{dt}{\sqrt{1-t^2}}$$

$$= -\sin^{-1} t + c_2$$

$$= -\sin^{-1}\left(\frac{a}{x}\right) + c_2.$$

$$I = I_1 + I_2$$

$$= \log\left|x + \sqrt{x^2 - a^2}\right| + c_1 - \sin^{-1}\left(\frac{a}{x}\right) + c_2$$

$$= \log\left|x + \sqrt{x^2 - a^2}\right| - \sin^{-1}\frac{a}{x} + c,$$

when $c = c_1 + c_2$.

Answer : $\int \frac{1}{x}\sqrt{\frac{x+a}{x-a}}\, dx = \log\left|x + \sqrt{x^2-a^2}\right| - \sin^{-1}\left(\frac{a}{x}\right) + c$

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SECTION-B

Q.21: (a)

let vector $\vec{a}$ make angles θ with all the three axes.

$\Rightarrow \cos^2\theta + \cos^2\theta + \cos^2\theta = 1$

$\Rightarrow 3\cos^2\theta = 1$

$\Rightarrow \cos\theta = \pm \frac{1}{\sqrt{3}}$.

Direction cosines of $\vec{a} = \pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Unit vector along the direction of $\vec{a} = \hat{a} = \pm\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$

$= \pm\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$.

Given: magnitude of vector $\vec{a} = 5\sqrt{3}$.

$\therefore \vec{a} = |\vec{a}|\cdot\hat{a} = \pm\frac{5\sqrt{3}}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k}) = \pm 5(\hat{i}+\hat{j}+\hat{k})$.

Answer: The required vectors are:

$\vec{a} = \pm 5(\hat{i}+\hat{j}+\hat{k})$.

Space for writing Question Number

Q.22 :

let $I = \int_0^{\pi/4} \sqrt{1+\sin 2x}\, dx$

$= \int_0^{\pi/4} \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}\, dx$

$= \int_0^{\pi/4} \sqrt{(\sin x + \cos x)^2}\, dx$

$= \int_0^{\pi/4} (\sin x + \cos x)\, dx$

$= \left[-\cos x + \sin x\right]_0^{\pi/4}$

$= \left[\left(-\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right) - \left(-\cos 0 + \sin 0\right)\right]$

$= \left[\left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (-1+0)\right]$

$= 1.$

Answer: $\int_0^{\pi/4} \sqrt{1+\sin 2x}\, dx = 1.$

Space for writing Question Number

Q. 23 :

$f(x) = \sin x - ax + b$

$f'(x) = \cos x - a$

For f to be increasing on R :

$f'(x) \geq 0$

$\cos x - a \geq 0$

$a \leq \cos x$.

Since the maximum value of $\cos x = 1$,

$a \in (-\infty, 1]$.

$\therefore$ a can take the values from set $(-\infty, 1]$ for $f(x)$ to be increasing on R.

Answer: $a \in (-\infty, 1]$

$[-1, 1]$ $\{0, 1\}$

$\cos x - 0 > 0$.

$\cos x - a = 0$

$\cos x = a$

$x = \cos^{-1} a$.

-1 1

$(-\infty, 1)$ $(-\infty, 1]$

$a \leq \cos x$

$\cos x - a \geq 0$

$a < \cos x$

-1

$a < 1$

$0, 1$

Space for writing Question Number

Q.24 :

$$\vec{\alpha} = (x-2)\vec{a} + \vec{b}$$
$$\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$$

PTO → for the answer.

~~let $\vec{a} = p_1\hat{i} + q_1\hat{j} + r_1\hat{k}$~~
~~& $\vec{b} = p_2\hat{i} + q_2\hat{j} + r_2\hat{k}$~~

~~$\vec{\alpha} = (x-2)(p_1\hat{i} + q_1\hat{j} + r_1\hat{k}) + p_2\hat{i} + q_2\hat{j} + r_2\hat{k}$~~
~~$= (xp_1 - 2p_1 + p_2)\hat{i} + (xq_1 - 2q_1 + q_2)\hat{j} + (xr_1 - 2r_1 + r_2)\hat{k}$~~

~~$\vec{\beta} = (3+2x)(p_1\hat{i} + q_1\hat{j} + r_1\hat{k}) - 2(p_2\hat{i} + q_2\hat{j} + r_2\hat{k})$~~
~~$= (3p_1 + 2xp_1 - 2p_2)\hat{i} + (3q_1 + 2xq_1 - 2q_2)\hat{j} + (3r_1 + 2xr_1 - 2r_2)$~~

~~Since~~

PTO →

Space for writing Question Number

Q.24 e

Since $\vec{\alpha}$ & $\vec{\beta}$ are ~~paral~~ collinear:

$$\vec{\alpha} = k\vec{\beta}$$

$$(x-2)\vec{a} + \vec{b} = k[(3+2x)\vec{a} - 2\vec{b}]$$

$$x\vec{a} - 2\vec{a} + \vec{b} = 3k\vec{a} + 2kx\vec{a} - 2k\vec{b}$$

$$(x-2)\vec{a} + \vec{b} = (3k+2kx)\vec{a} - 2k\vec{b}$$

$$\cancel{-2k = 1}$$

$$\cancel{k = \frac{-1}{2}}$$

$$-2k = 1 \Rightarrow k = -\frac{1}{2}$$

$$3k + 2kx = x - 2$$

$$\frac{-3}{2} - x = x - 2$$

$$2x = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\therefore \quad x = \frac{1}{4}$$

02

Answer: $x = 1/4$

Space for writing Question Number

Q.23:
(a)

Given: $e^{x/y} = x$

To prove: $\dfrac{dy}{dx} = \dfrac{x-y}{x\log x}$.

$\dfrac{x-y}{x \cdot \frac{x}{y}}$

$\dfrac{\frac{xy-y^2}{y}}{x^2}$

Proof: $e^{x/y} = x$

$\log\left(e^{\frac{x}{y}}\right) = \log x$

$\dfrac{x}{y} = \log x$.

Differentiating both ~~sides~~ sides w.r.t. x:

$\dfrac{1}{y^2}\left(y - \dfrac{x\,dy}{dx}\right) = \dfrac{1}{x}$ ⇒ PTO →

$\cancel{\dfrac{x\,dy}{dx} = y - \dfrac{1}{x} = \dfrac{xy-1}{x}}$

$\cancel{\dfrac{dy}{dx} = \dfrac{xy-1}{x^2}}$

Space for writing Question Number

$$y - x\frac{dy}{dx} = \frac{y^2}{x}$$

$$\frac{x\,dy}{dx} = y - \frac{y^2}{x} = \frac{xy - y^2}{x}$$

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2}$$

LHS: $\frac{dy}{dx} = \frac{xy - y^2}{x^2}$,

RHS: $\frac{x-y}{x\log x} = \frac{x-y}{x\left(\frac{x}{y}\right)} = \frac{xy - y^2}{x^2}$,

$\therefore$ LHS = RHS

Hence, proved.

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Space for writing Question Number

$\boxed{\text{ROUGH}}$

$(A+B)(A+B)$

$A^2 + AB + BA$ $2x-4=0$

$+B^2$ $x=2$

$AB = BA$

$\dfrac{P(\bar{E} \cap \bar{F})}{P(\bar{F})}$

$\dfrac{P((E \cup F)')}{P(\bar{F})}$

$\dfrac{x-1}{5} = y+5 = \dfrac{z+6}{3}$

$1, -5, -6$

$\dfrac{x-4}{} = \dfrac{y+3}{1} = \dfrac{z-7}{2}$

$x\sqrt{1+y} = -y\sqrt{1+x}$

$x^2(1+y) = y^2(1+x)$

$y = 2x^3$

$3\left(\dfrac{dy}{dx}\right)^2 \dfrac{dy}{dx} = 0$

$\begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & \end{bmatrix} = \begin{bmatrix} 1 & 6x & 8x \\ 12 & 5 & 4 \\ 4y & 22 & 6 \end{bmatrix}$

$3t+4$

$t-3$

$2t+7$

$x=2$ $y=4$

$2x-4$

$4y = 8x$

$x=2$ $4+4$ $4+4=8$

$4AB + 3AB + 3BA - 4BA$

$7AB - BA$

$\dfrac{1}{2} + \dfrac{1}{4} + \cos\theta$ $\dfrac{dy}{} = 2x$

$A+B=1$

$B = \dfrac{2}{5}$

$\dfrac{1}{2} - \dfrac{1}{4} = \cos\theta$

$\dfrac{\pi}{4}$

$5A = 3$

$A = \dfrac{3}{5}$

$\dfrac{dv}{dt}$ $100\pi \dfrac{dh}{dt} = 100\pi$

$\dfrac{1}{4}$ $= \dfrac{1}{2}$

$|p+q|^2 = 1+1+2pq = 1$

$pq = -\dfrac{1}{2}$

$\cos\theta = -\dfrac{1}{2}$

$2C + A$