CBSE CLASS X MATHEMATICS STANDARD (041)

SAMPLE QUESTION PAPER 2026–27

COMPLETE SOLUTIONS

SECTION A

Question 1 [1marks]

The greatest number which divides both 134 and 188, leaving remainders 4 and 6 respectively.

Let the required number be d.
Then d divides (134 − 4) = 130 and (188 − 6) = 182.
Therefore,
d = HCF(130, 182)
130 = 2 × 5 × 13
182 = 2 × 7 × 13
Hence, HCF = 2 × 13 = 26.

Answer: 26 (Option B)

Question 2 [1 marks]

If $f(x) = px^2 + qx + r$, $p \neq 0$ and $p + r = q$, find one zero of f(x).

Given p + r = q.
Put x = −r/p:
$f(-r/p) = p(r^2/p^2) - qr/p + r$
= (r/p)(r − q + p).
Since p + r = q, r − q + p = 0.
Therefore f(−r/p) = 0.

Answer: −r/p (Option C)

Question 3 [1 marks]

Tarun found the intersection of two lines as (5, −1). One line is x − y = 6. Which listed line could be the other line?

I. $3x - 3y = 18$

Divide by 3:

$$x - y = 6$$

This is **exactly the same line** as the first line.

Therefore, the two equations have **infinitely many common points**, not only $(5, -1)$.

❌ **I is not possible.**

II. $2x - 3y = 13$

Substitute $(5, -1)$:

$$2(5) - 3(-1) = 10 + 3 = 13$$

So $(5, -1)$lies on this line.

It is **not the same line** as $x - y = 6$, so the two lines intersect at exactly one point.

✅ **II is possible.**

III. $2x - 3y = 16$

Substitute $(5, -1)$:

$$2(5) - 3(-1) = 10 + 3 = 13 \neq 16$$

So $(5, -1)$does **not** lie on this line.

❌ **III is not possible.**

Answer: II Only(Option B)

Question 4 [1 marks]

If (1 – p) is a root of $x^2 + px + 1 - p = 0$, find its roots.

Since x = 1 – p is a root,
$(1 - p)^2 + p(1 - p) + 1 - p = 0$
$\Rightarrow 2(1 - p) = 0$
$\Rightarrow p = 1.$

The equation becomes $x^2 + x = 0$
$\Rightarrow x(x + 1) = 0$.

Answer: 0 and −1 (Option C)

Question 5 [1 marks]

Find the middle term of the A.P. 10, 7, 4, …, −62.

Here $a = 10$, $d = -3$.
$-62 = 10 + (n - 1)(-3)$
$\Rightarrow -72 = -3(n - 1)$
$\Rightarrow n - 1 = 24$
$\Rightarrow n = 25$.

Since n is odd, the middle term is the 13th term.
$a_{13} = 10 + 12(-3) = -26$.

Answer: −26 (Option A)

Question 6 [1 marks]

The perimeters of two similar triangles are 56 cm and 70 cm. If a side of the first triangle is 14 cm, find the corresponding side of the second triangle.

For similar triangles, the ratio of corresponding sides equals the ratio of their perimeters.

Second side / 14 = 70 / 56 = 5/4
Therefore, second side = 14 × 5/4 = 17.5 cm.

Answer: 17.5 cm (Option D)

Question 7 [1 marks]

Find the point lying on the perpendicular bisector of the segment joining A(−3, −4) and B(3, 4).

The midpoint of AB is
$((-3 + 3)/2, (-4 + 4)/2) = (0, 0)$.

The perpendicular bisector passes through the midpoint.

Answer: (0, 0) (Option A)

Question 8 [1 marks]

If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = 1/\sqrt{3}$, with $0° < A + B < 90°$ and $A > B$, find A and B.

Since $0° < A + B < 90°$ and $\tan(A + B) = \sqrt{3}$,
$A + B = 60°$.

Also, $A > B$ and $\tan(A - B) = 1/\sqrt{3}$, so
$A - B = 30°$.

Adding:
$2A = 90° \Rightarrow A = 45°$.
Hence $B = 15°$.

Answer: A = 45°, B = 15° (Option C)

Question 9 [1 marks]

T-shirts numbered 4 to 99 are placed in a box. Find the probability of getting a number that is a perfect square or a perfect cube.

Total numbers = 99 − 4 + 1 = 96.

Perfect squares in this range:
4, 9, 16, 25, 36, 49, 64, 81 → 8 numbers.

Perfect cubes in this range:
8, 27, 64 → 3 numbers.

64 is common to both sets, so favourable numbers
= 8 + 3 − 1 = 10.

Probability = 10/96 = 5/48.

Answer: 5/48 (Option D)

Question 10 [1 marks]

A car wheel has diameter 21 cm. Find the number of complete revolutions in moving 66 km.

Circumference of wheel = $\pi d = (22/7) \times 21 = 66$ cm.

66 km = $66 \times 1000 \times 100 = 6{,}600{,}000$ cm.

Number of revolutions = $6{,}600{,}000 / 66 = 100{,}000 = 10^5$.

Answer: 10^5 (Option B)

Question 11 [1 marks]

Two cubes, each of volume 64 cm^3, are joined end to end. Find the surface area of the resulting cuboid.

For each cube, $side^3 = 64 \Rightarrow side = 4$ cm.

The resulting cuboid has dimensions 8 cm × 4 cm × 4 cm.

Surface area = $2(lb + bh + hl)$
= $2(8\times4 + 4\times4 + 8\times4)$
= $2(32 + 16 + 32)$
= 160 cm^2.

Answer: 160 cm^2 (Option B)

Question 12 [1 marks]

The mean age of a combined group of men and women is 35 years. The mean ages of men and women are 38 years and 30 years respectively. Find the percentage of women.

Let the numbers of men and women be m and w.

$38m + 30w = 35(m + w)$
$\Rightarrow 3m = 5w$
$\Rightarrow m:w = 5:3$.

Percentage of women = $3/(5+3) \times 100 = 37.5\%$.

Answer: 37.5% (Option D)

Question 13 [1 marks]

Two dice are rolled simultaneously. Find the probability of getting a number less than 4 on each die.

For one die, favourable outcomes are 1, 2, 3.
$P(\text{number} < 4) = 3/6 = 1/2$.

For two independent dice:
$P = 1/2 \times 1/2 = 1/4$.

Answer: 1/4 (Option A)

Question 14 [1 marks]

PM is a median of ΔPQR with $P(5, -6)$, $Q(6, 4)$, $R(0, 0)$. Find PM.

Since PM is a median, M is the midpoint of QR.

$M = ((6+0)/2, (4+0)/2) = (3, 2)$.

$PM = \sqrt{[(5-3)^2 + (-6-2)^2]}$
$= \sqrt{(4 + 64)}$
$= \sqrt{68}$
$= 2\sqrt{17}$ units.

Answer: $2\sqrt{17}$ units (Option A)

Question 15 [1 marks]

An arc of a circle has length 6π cm and the sector it bounds has area 24π cm². Find the radius.

Let radius be r and angle be θ radians.

Arc length $= r\theta = 6\pi$.
Sector area $= 1/2\ r^2\theta = 24\pi$.

Using $r\theta = 6\pi$:
Sector area $= 1/2\ r(6\pi) = 3\pi r$.

Thus $3\pi r = 24\pi \Rightarrow r = 8$ cm.

Answer: 8 cm (Option B)

Question 16 [1 marks]

The mean and median of a data set are 35.5 and 32 respectively. Find the mode.

Using the empirical relation:
Mode = 3(Median) − 2(Mean)

= 3(32) − 2(35.5)
= 96 − 71
= 25.

Answer: 25 (Option C)

Question 17 [1 marks]

A kite string makes angle θ with the ground, tan θ = 12/5. If the string is 52 m long, find the height of the kite.

tan θ = perpendicular/base = 12/5.

Thus the corresponding right triangle has sides 12, 5, 13.
For hypotenuse 52 m, scale factor = 52/13 = 4.

Height = 12 × 4 = 48 m.

Answer: 48 m (Option C)

Question 18 [1 marks]

Two diamonds and four spades are missing from a pack of 52 cards. Find the probability of drawing a heart.

Cards remaining = 52 − 2 − 4 = 46.

All 13 hearts are still present.

Probability = 13/46.

Answer: 13/46 (Option B)

Question 19 [1 marks]

Assertion (A): If three vertices of a parallelogram taken in order are (−1, −6), (2, −5) and (7, 2), its fourth vertex is (4, 1).

Reason (R): Diagonals of a parallelogram bisect each other.

Let the vertices be $A(-1,-6)$, $B(2,-5)$, $C(7,2)$, $D(x,y)$.
Since diagonals bisect each other, midpoint of AC = midpoint of BD.

Midpoint of AC = $((-1+7)/2, (-6+2)/2) = (3,-2)$.

Therefore,
$((2+x)/2, (-5+y)/2) = (3,-2)$
$\Rightarrow x = 4, y = 1$.

Thus the Assertion is true. The Reason is also true and correctly explains the result.

Answer: Option A — Both Assertion and Reason are true, and Reason is the correct explanation.

Question 20 [1 marks]

Assertion (A): If the zeroes of $(2k - 1)x^2 + 4x - 3$ are reciprocal of each other, then $k = -1$.

Reason (R): If $a = c$, then the zeroes of $ax^2 + bx + c$, $a \neq 0$, are reciprocal of each other.

For $ax^2 + bx + c$, the product of zeroes is c/a.
If the zeroes are reciprocal, their product is 1.

Here,
$(-3)/(2k-1) = 1$
$\Rightarrow -3 = 2k - 1$
$\Rightarrow 2k = -2$
$\Rightarrow k = -1$.

The Reason is true because when $a = c$, $c/a = 1$, so the product of the zeroes is 1.

Answer: Option A — Both Assertion and Reason are true, and Reason is the correct explanation.

SECTION B — Very Short Answer Questions

Question 21(A) [2 marks]

A hall has length 9.75 m, breadth 6.75 m and height 5.25 m. Find the length of the longest unmarked ruler that can exactly measure all three dimensions.

Convert to centimetres:
9.75 m = 975 cm, 6.75 m = 675 cm, 5.25 m = 525 cm.

The required ruler length is HCF(975, 675, 525).

HCF(975,675) = 75 and HCF(75,525) = 75.

Answer: 75 cm (or 0.75 m).

Question 21(B) [2 marks]

Three bells toll at intervals of 5, 6 and 8 minutes. They toll together at 10:00 a.m. How many more times do they all toll together before the park closes at 7:00 p.m.?

They toll together every LCM(5,6,8) minutes.
LCM = 120 minutes = 2 hours.

After 10:00 a.m., they toll together at:
12:00 noon, 2:00 p.m., 4:00 p.m. and 6:00 p.m.

The next common toll would be 8:00 p.m., after closing.

Answer: 4 more times.

Question 22(A) [2 marks]

ABCD is a trapezium with $AB \parallel DC$ and diagonals intersecting at O. Show that $AO/BO = CO/DO$.

In ΔAOB and ΔCOD:

$\angle AOB = \angle COD$ (vertically opposite angles).

Also, $\angle ABO = \angle CDO$ (alternate interior angles, since $AB \parallel DC$).

Therefore, $\Delta AOB \sim \Delta COD$ by AA similarity.

Hence,
$AO/CO = BO/DO$.

Rearranging,
$AO/BO = CO/DO$.

Answer: $AO/BO = CO/DO$.

Question 22(B) [2 marks]

In the given figure, $\angle PQR = \angle QSP$, $PQ = 6$ cm and $PS = 3$ cm. Find PR.

Since S lies on PR,
$\angle QPS = \angle QPR$.

Given $\angle QSP = \angle PQR$.

Therefore, $\Delta QSP \sim \Delta PQR$ by AA similarity.

Corresponding sides give:
$PS/PQ = PQ/PR$.

Thus,
$3/6 = 6/PR$
$\Rightarrow PR = 36/3 = 12$ cm.

Answer: PR = 12 cm.

Visually Impaired Candidates — Alternative to Q22: In a right-angled triangle ABC, right-angled at B, a perpendicular BD is drawn to hypotenuse AC. Prove $\Delta BCD \sim \Delta ACB$.

$\angle BDC = 90° = \angle ABC$.
Also, $\angle BCD = \angle ACB$ because CD lies along CA.
Therefore, $\Delta BCD \sim \Delta ACB$ by AA similarity.

Question 23 [2 marks]

If $\tan\theta = 1/\sqrt{5}$, find $(\operatorname{cosec}^2\theta - \sec^2\theta)/(\operatorname{cosec}^2\theta + \sec^2\theta)$.

$\tan^2\theta = 1/5$.

Using $\sec^2\theta = 1 + \tan^2\theta$:
$\sec^2\theta = 6/5$.

Also,
$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + 5 = 6$.

Therefore,
$(\text{cosec}^2\theta - \sec^2\theta)/(\text{cosec}^2\theta + \sec^2\theta)$
$= (6 - 6/5)/(6 + 6/5)$
$= (24/5)/(36/5)$
$= 2/3.$

Answer: 2/3.

Question 24 [2 marks]

Find the area of a sector of a circle with diameter 28 cm if the corresponding arc length is 22 cm.

Radius $r = 28/2 = 14$ cm.

Arc length $l = 22$ cm.

Area of sector $= 1/2 \times r \times l$
$= 1/2 \times 14 \times 22$
$= 154\ \text{cm}^2$.

Answer: 154 cm^2.

Question 25 [2 marks]

Two dice are thrown simultaneously. Find the probability that the product of the numbers appearing on them is a prime number.

A product of two positive integers is prime only when one factor is 1 and the other factor is prime.

Prime numbers on a die are 2, 3 and 5.

Favourable ordered outcomes:
(1,2), (2,1), (1,3), (3,1), (1,5), (5,1).

Number of favourable outcomes = 6.
Total outcomes = 36.

Probability = 6/36 = 1/6.

Answer: 1/6.

SECTION C — Short Answer Questions

Question 26 [3 marks]

Given that √3 is irrational, prove that 2 + 3√3 is irrational.

Let 2 + 3√3 is a rational number.

Then

$2 + 3\sqrt{3} = \frac{a}{b}$, where 'a' and 'b' are integers & b ≠ 0

$$2 + 3\sqrt{3} = \frac{a}{b}$$

$$3\sqrt{3} = \frac{a}{b} - 2$$

$$\sqrt{3} = \frac{a-2b}{3b}$$

Since 'a' and 'b' are integers.

$\frac{a-2b}{3b}$ is a rational number.

This contradicts the given fact that √3 is irrational.

Hence, our assumption is wrong. Therefore, 2 + 3√3 is irrational.

Answer: 2 + 3√3 is irrational.

Question 27 [3 marks]

If α and β are the zeroes of $2x^2 - 8x + 5$, find (α + 1/β)(β + 1/α).

For $2x^2 - 8x + 5$,
α + β = 8/2 = 4,
αβ = 5/2.

Now,
$(\alpha + 1/\beta)(\beta + 1/\alpha)$
$= \alpha\beta + 1 + 1 + 1/(\alpha\beta)$
$= \alpha\beta + 2 + 1/(\alpha\beta)$.

$= 5/2 + 2 + 2/5$
$= (25 + 20 + 4)/10$
$= 49/10$.

Answer: 49/10.

Question 28(A) [3 marks]

A polygon has n sides. Its smallest exterior angle is 8° and each subsequent exterior angle is 4° more than the previous one. Find n.

The exterior angles form an A.P.:
8°, 12°, 16°, …, with common difference 4°.

Let the number of sides be n. Since the sum of exterior angles of any polygon is 360°,

$n/2\ [2(8) + (n-1)4] = 360$
$\Rightarrow n/2\ [16 + 4n - 4] = 360$
$\Rightarrow n/2\ (4n + 12) = 360$
$\Rightarrow n(n + 3) = 180$
$\Rightarrow n^2 + 3n - 180 = 0$
$\Rightarrow (n + 15)(n - 12) = 0$.

Since n is positive, n = 12.

Answer: 12 sides.

Question 28(B) [3 marks]

Find the sum of integers between 1 and 400 that are multiples of both 4 and 5.

A number that is divisible by both 4 and 5 must be divisible by:

$$\mathrm{LCM}(4,5) = 20$$

Therefore, the required A.P. is:

$$20,40,60,\ldots,380$$

Here,

$$a = 20,\ d = 20,\ a_n = 380$$

Using

we get

$$380 = 20 + (n-1)20$$
$$360 = 20(n-1)$$
$$n - 1 = 18$$
$$\boxed{n = 19}$$

Now,

$$S_n = \frac{n}{2}(a + l)$$
$$S_{19} = \frac{19}{2}(20 + 380)$$
$$= \frac{19}{2} \times 400$$
$$= 19 \times 200 = 3800$$

Answer: 3800

Question 29 [3 marks]

Prove that a parallelogram circumscribing a circle is a rhombus.

Let ABCD be a parallelogram circumscribing a circle.

For any quadrilateral circumscribing a circle,
AB + CD = BC + DA.

Since ABCD is a parallelogram,
AB = CD and BC = DA.

Therefore,
AB + AB = BC + BC

⇒ 2AB = 2BC
⇒ AB = BC.

A parallelogram with two adjacent sides equal is a rhombus.

Hence, ABCD is a rhombus.

Answer: The parallelogram is a rhombus.

Question 30 [3 marks]

In what ratio does the x-axis divide the line segment joining (−4, −6) and (−1, 7)? Find the point of division.

Given points:

$$A(-,4-6),\ B(-,17)$$

The **x-axis** has $y=0$. Suppose it divides ABin the ratio $m:n$.

Step 1: Apply the section formula

If a point Pdivides $A(x_1,y_1)$and $B(x_2,y_2)$in the ratio $m:n$, then

$$y=\frac{my_2+ny_1}{m+n}$$

Since Plies on the x-axis,

$$y=0$$

Therefore,

$$0=\frac{m(7)+n(-6)}{m+n}$$
$$7m-6n=0$$
$$7m=6n$$

Hence,

$$\boxed{m:n=6:7}$$

So the x-axis divides the line segment internally in the ratio

$$\boxed{6:7}$$

Step 2: Find the coordinates of the point

Using the section formula for the x-coordinate:

$$x = \frac{mx_2 + nx_1}{m+n}$$

Putting $m = 6, n = 7$:

$$x = \frac{6(-1) + 7(-4)}{6+7}$$
$$= \frac{-6-28}{13}$$
$$= -\frac{34}{13}$$

And since the point lies on the x-axis,

$$y = 0$$

Therefore, the point of division is

Answer: Ratio = 6:7; point of division = (−34/13, 0).

Question 31(A) [3 marks]

Simplify: cos θ/(1 − sin θ) + (1 − sin θ)/cos θ. Identify Jyoti's error and rectify it.

Jyoti's Step 1 is correct:
cosθ/(1−sinθ) + (1−sinθ)/cosθ
= [cos²θ + (1−sinθ)²]/[(1−sinθ)cosθ].

The error occurs in Step 2. Jyoti replaced (1−sinθ)² by cos²θ, which is not a valid identity.

Correctly,
cos²θ + (1−sinθ)²
= cos²θ + 1 − 2sinθ + sin²θ

$= 1 + 1 - 2\sin\theta$
$= 2(1 - \sin\theta)$.

Therefore,
$[2(1 - \sin\theta)]/[(1 - \sin\theta)\cos\theta]$
$= 2/\cos\theta$
$= 2 \sec\theta$.

Answer: Error: Step 2. Correct value = $2 \sec \theta$.

Question 31(B) [3 marks]

If $1/(\sin x - \cos x) = \operatorname{cosec} x/\sqrt{2}$, prove that $(1/(\sin x + \cos x))^2 = \sec^2 x/2$.

Given:
$1/(\sin x - \cos x) = \operatorname{cosec} x/\sqrt{2}$.

Taking reciprocals,
$\sin x - \cos x = \sqrt{2} \sin x$.

Squaring,
$(\sin x - \cos x)^2 = 2\sin^2 x$.

Now,
$(\sin x + \cos x)^2$
$= \sin^2 x + \cos^2 x + 2\sin x \cos x$.

From the given relation:
$\sin x - \cos x = \sqrt{2} \sin x$.
Squaring:
$\sin^2 x + \cos^2 x - 2\sin x \cos x = 2\sin^2 x$.
Since $\sin^2 x + \cos^2 x = 1$,
$1 - 2\sin x \cos x = 2\sin^2 x$,
so $1 = 2\sin x(\sin x + \cos x)$.

Hence,
$\sin x + \cos x = 1/(2\sin x)$.

A direct reciprocal-square form can also be obtained more cleanly by adding the identity

$(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$.
Using $(\sin x - \cos x)^2 = 2\sin^2 x$:
$(\sin x + \cos x)^2 = 2 - 2\sin^2 x = 2\cos^2 x$.

Therefore,
$1/(\sin x + \cos x)^2 = 1/(2\cos^2 x) = \sec^2 x/2$.

Answer: $(1/(\sin x + \cos x))^2 = \sec^2 x/2$.

SECTION D — Long Answer Questions

Question 32 [5 marks]

A rectangular park is 50 m × 40 m. A rectangular swimming pool is constructed in the middle, surrounded by a uniform-width grass strip. The grass area is 1184 m². Find the dimensions of the pool.

Let the uniform width of the grass strip be x m.

Then the pool dimensions are:
$(50 - 2x)$ m and $(40 - 2x)$ m.

Area of park = $50 \times 40 = 2000$ m².
Area of pool = $2000 - 1184 = 816$ m².

Therefore,
$(50 - 2x)(40 - 2x) = 816$.

$2000 - 180x + 4x^2 = 816$
$\Rightarrow 4x^2 - 180x + 1184 = 0$
$\Rightarrow x^2 - 45x + 296 = 0$
$\Rightarrow (x - 8)(x - 37) = 0$.

Thus x = 8 m or 37 m.

Since the strip width cannot be 37 m (it would make the pool dimensions negative), x = 8 m.

Pool length = $50 - 16 = 34$ m.
Pool breadth = $40 - 16 = 24$ m.

Answer: Dimensions of swimming pool = 34 m × 24 m.

Question 33 [5 marks]

Prove the Basic Proportionality Theorem: a line drawn parallel to one side of a triangle divides the other two sides in the same ratio. Then apply it to trapezium PQRS with PQ ∥ SR and XY ∥ PQ, where X and Y lie on PS and QR respectively, to prove PX/XS = QY/YR.

Basic Proportionality Theorem (BPT):

In ΔABC, let DE ∥ BC, where D lies on AB and E lies on AC.

Since DE ∥ BC,
∠ADE = ∠ABC and ∠AED = ∠ACB.
Therefore, ΔADE ~ ΔABC.

Hence,
AD/AB = AE/AC.

Using AB = AD + DB and AC = AE + EC, this gives
AD/(AD+DB) = AE/(AE+EC).

Cross-multiplication:
AD(AE+EC) = AE(AD+DB)
⇒ AD·EC = AE·DB
⇒ AD/DB = AE/EC.

Thus a line parallel to one side of a triangle divides the other two sides in the same ratio.

Application to trapezium:

Since PQ ∥ XY ∥ SR, consider the two transversals PS and QR between the parallel lines.

By BPT (or by applying the theorem in the triangle formed by extending the non-parallel sides),
PX/XS = QY/YR.

Answer: PX/XS = QY/YR.

Question 34(A) [5 marks]

A tent is a cylinder surmounted by a cone. Cylindrical height = 3 m, radius = 14 m, total height = 13.5 m. Find the canvas required, allowing 26 m^2 for stitching and wastage, and its cost at ₹250 per m^2.

Radius r = 14 m.
Height of cylindrical part = 3 m.
Total height = 13.5 m.

Height of cone = 13.5 − 3 = 10.5 m.

Slant height of cone:
$l = \sqrt{(r^2 + h^2)}$
$= \sqrt{(14^2 + 10.5^2)}$
$= \sqrt{(196 + 110.25)}$
$= \sqrt{306.25}$
= 17.5 m.

Canvas required for the tent consists of the curved surface of the cylinder and curved surface of the cone.

CSA of cylinder = $2\pi rh$
$= 2 \times 22/7 \times 14 \times 3$
= 264 m^2.

CSA of cone = πrl
$= 22/7 \times 14 \times 17.5$
= 770 m^2.

Total canvas for tent = 264 + 770 = 1034 m^2.

Including 26 m^2 for stitching and wastage:
Total canvas to be purchased = 1034 + 26 = 1060 m^2.

Cost = 1060 × ₹250 = ₹2,65,000.

Answer: Canvas = 1060 m^2; Cost = ₹2,65,000.

Question 34(B) [5 marks]

A solid wooden toy is a hemisphere surmounted by a cone of the same radius. Radius = 3.5 cm and total volume = 166 5/6 cm^3. Find the height of the conical part and the cost of painting the hemispherical part at ₹15 per cm^2.

r = 3.5 cm = 7/2 cm.
Total volume = 166 5/6 = 1001/6 cm^3.

Volume of hemisphere = $(2/3)\pi r^3$.
Volume of cone = $(1/3)\pi r^2 h$.

Therefore,
$(2/3)\pi r^3 + (1/3)\pi r^2 h = 1001/6$.

Using $\pi = 22/7$ and $r = 7/2$:

$(2/3)(22/7)(7/2)^3 + (1/3)(22/7)(7/2)^2 h = 1001/6$.

Solving gives h = 6 cm.

Curved surface area of hemisphere:
CSA = $2\pi r^2$
= $2 \times 22/7 \times (7/2)^2$
= 77 cm^2.

Cost of painting = 77 × ₹15 = ₹1155.

Answer: Height of cone = 6 cm; Painting cost = ₹1155.

Question 35(A) [5 marks]

Manish observes a kite 200 m away at an angle of elevation 30°. Mahesh stands on the roof of a 50 m high building on the opposite side of the kite and observes it at 45°. Find the distance between the kite and Mahesh.

Let the height of the kite above the ground be h.

For Manish:
$\tan 30° = h/200$
$\Rightarrow 1/\sqrt{3} = h/200$

$\Rightarrow h = 200/\sqrt{3}$ m.

Mahesh is on a 50 m high roof. Let the horizontal distance between Mahesh's building and the vertical projection of the kite be x.

Since angle of elevation is 45°:
$\tan 45° = (h - 50)/x$
$\Rightarrow 1 = (h - 50)/x$
$\Rightarrow x = h - 50$.

The required distance d between Mahesh and the kite is the hypotenuse:
$d = (h - 50)/\sin 45°$
$= \sqrt{2}(h - 50)$
$= \sqrt{2}(200/\sqrt{3} - 50)$.

Numerically,
$h \approx 115.47$ m,
$x \approx 65.47$ m,
$d \approx 92.59$ m.

Answer: Distance between the kite and Mahesh ≈ 92.59 m.

Question 35(B) [5 marks]

The angles of depression of two ships from the top of a lighthouse on the same side are 45° and 30°. The ships are 200 m apart and one is directly behind the other. Find the height of the lighthouse.

Let the height of the lighthouse be h m.

For the nearer ship:
$\tan 45° = h/x$
$\Rightarrow x = h$.

For the farther ship, let its distance from the foot of the lighthouse be y:
$\tan 30° = h/y$
$\Rightarrow 1/\sqrt{3} = h/y$
$\Rightarrow y = h\sqrt{3}$.

The ships are 200 m apart:
$y - x = 200$
$\Rightarrow h\sqrt{3} - h = 200$

$\Rightarrow h(\sqrt{3} - 1) = 200$.

Therefore,
$h = 200/(\sqrt{3} - 1)$
$= 200(\sqrt{3} + 1)/(3 - 1)$
$= 100(\sqrt{3} + 1)$ m.

Answer: Height of lighthouse = $100(\sqrt{3} + 1)$ m ≈ 273.21 m.

SECTION E — Case Study Based Questions

Question 36 [4 marks]

A company manufactures small and large sanitizers. In June, small bottles cost ₹10 and large bottles cost ₹15; 1000 bottles were sold for ₹12,750. In July, both prices increased by ₹2; 2500 bottles were sold for ₹34,250.

(i) Form the linear equation for June.

Let x = number of small bottles and y = number of large bottles sold.
Total bottles: $x + y = 1000$.
Total sale: $10x + 15y = 12,750$.

Answer: June equations: $x + y = 1000$; $10x + 15y = 12,750$.

(ii) Form the linear equation for July.

New prices are ₹12 and ₹17.
Total bottles: $x + y = 2500$.
Total sale: $12x + 17y = 34,250$.

Answer: July equations: $x + y = 2500$; $12x + 17y = 34,250$.

(iii-A) How many sanitizer bottles of each type were sold in June?

$x + y = 1000$
$10x + 15y = 12,750$.

Multiplying the first equation by 10:
$10x + 10y = 10,000$.

Subtracting:
$5y = 2,750$

⇒ y = 550.

Hence x = 1000 − 550 = 450.

Answer: June: 450 small bottles and 550 large bottles.

(iii-B) How many sanitizer bottles of each type were sold in July?

x + y = 2500
12x + 17y = 34,250.

Multiplying the first equation by 12:
12x + 12y = 30,000.

Subtracting:
5y = 4,250
⇒ y = 850.

Hence x = 2500 − 850 = 1650.

Answer: July: 1650 small bottles and 850 large bottles.

Question 37 [4 marks]

A circular meditation garden has radius 9 m. From an external point P, two tangents PA and PB are drawn. OP = 18 m. Answer the following.

(i) What is the measure of ∠OAP?

A radius drawn to the point of contact of a tangent is perpendicular to the tangent.

Answer: ∠OAP = 90°.

(ii) Calculate PA.

In right ΔOAP:
OP = 18 m, OA = 9 m.

By Pythagoras:
$PA^2 = OP^2 - OA^2$
$= 18^2 - 9^2$
= 324 − 81
= 243.

Therefore PA = $\sqrt{243} = 9\sqrt{3}$ m.

Answer: PA = $9\sqrt{3}$ m.

(iii-A) Calculate ∠AOB.

In right ΔOAP:
OA/OP = 9/18 = 1/2.

Thus ∠OPA = 30°, so ∠AOP = 60°.

Tangents from an external point are equal, and OP bisects the angle between the two tangents. Therefore:
∠AOB = 2∠AOP = 120°.

Answer: ∠AOB = 120°.

(iii-B) If PA and PB were inclined at 60°, what would be their length?

If ∠APB = 60°, OP bisects it, so ∠APO = 30°.

In right ΔOAP:
OA = 9 m and ∠APO = 30°.

tan 30° = OA/PA = $1/\sqrt{3}$.

Therefore,
PA = $9\sqrt{3}$ m.

Answer: Each tangent would have length $9\sqrt{3}$ m.

Question 38 [4 marks]

Marks obtained by Class X students are grouped as 0–10, 10–20, 20–30, 30–40, 40–50 with frequencies 3, 6, 12, 15, 14 respectively.

(i) Identify the class with the most common performance range.

The highest frequency is 15, corresponding to the class interval 30–40.

Answer: 30–40.

(ii) Find the class interval containing the median.

Total frequency N = 3 + 6 + 12 + 15 + 14 = 50.
N/2 = 25.

Cumulative frequencies are:
0–10: 3
10–20: 9
20–30: 21
30–40: 36
40–50: 50.

The first cumulative frequency greater than 25 is 36, corresponding to 30–40.

Answer: Median class = 30–40.

(iii-A) Find the average performance of the students.

Class marks are 5, 15, 25, 35 and 45.

C.I.	f_i	x_i	f_ix_i
0 – 10	3	5	15
10 – 20	6	15	90
20 – 30	12	25	300
30 – 40	15	35	525
40 – 50	14	45	630
Total	50		1560

Mean = $\Sigma f_i x_i / \Sigma f_i$
= 1560/50
= 31.2.

Answer: Average performance = 31.2 marks.

(iii-B) Find the mode of the data.

The modal class is 30–40 because it has the highest frequency.

For grouped data:

$$\text{Mode} = l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)]h$$

Here,

$l = 30, f_1 = 15, f_0 = 12, f_2 = 14, h = 10.$

$$\text{Mode} = 30 + [(15-12)/(30-12-14)]\times 10$$
$$= 30 + (3/4)\times 10$$
$$= 30 + 7.5$$
$$= 37.5.$$

Answer: Mode = 37.5 marks.